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TWO-STREAM INSTABILITY FOR A SCATTERED BEAM PROPAGATING IN A COLLISIONAL PLASMA

by

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1. INTRODUCTION

It has long been appreciated that the high-frequency ($\omega \approx \omega_p$) two-stream instability can be stabilized if the background plasma is sufficiently collisional and the beam has a finite velocity spread.[1] With a renewed interest in relativistic electron beams for plasma heating[2] and in order to interpret experiments on the propagation of these beams through neutral gases at low pressures,[3,4] we have been investigating this problem anew.

The early calculations were essentially nonrelativistic with relativistic effects modeled in an ad hoc way through introduction of an anisotropic mass. Phenomenological models used to interpret neutral gas propagation experiments[3] are essentially equivalent to the early, nonrelativistic, Vlasov calculations. In order to put these models in perspective and to provide a linear theory from which nonlinear saturation amplitudes can be obtained by partial numerical simulation techniques,[5] we have been carrying out a rigorous Vlasov theory of the collisional stabilization of the two-stream instability in a relativistic electron beam. The equilibrium we choose is that of the so-called "scattered" beam[2,6] for which the electrons are monochromatic in energy but which have a distribution in the directions of their momenta. This distribution is characteristic of that produced by scattering in an anode foil and, in any case, the qualitative results are likely to be typical of those for other reasonable choices of relativistic distribution function. The stability of pinched beams can be estimated by taking the mean scattering angle, $\bar{\theta}$, which characterizes the distribution function to be equal to the betatron angle, $(I_B/2I_A)^{\frac{1}{2}}$ where I_B is the beam current and I_A the Alfven current. The beam and plasma are otherwise homogeneous and infinite in extent.

The linear dispersion relation is described in Sec. II along with the lowest order correction due to momentum scatter to the cold beam, one-dimensional dispersion relation. Some numerical results are presented in Sec. III and a

II. LINEAR DISPERSION RELATION

In this section, the linear dispersion relation for electrostatic waves in the beam-plasma system is presented.

The plasma is characterized by its plasma frequency ω_p and a (constant) collision frequency ν, measured in units of ω_p as are all frequencies below. The beam is characterized by a mean momentum P_0 (or energy $\gamma_0 = (1 + P_0^2)^{1/2}$ in units of the electron rest momentum m_0c and a mean scattering angle $\bar{\theta}$. The beam distribution function[6] is taken to be

$$f(\vec{p}) = \frac{a}{4\pi P_0^2}\,\delta(p - P_0)\mathrm{csch}(a)\exp(a\cos\theta) \tag{1}$$

where $a = 2/\bar{\theta}^2$.

The dispersion relation is $1 = \varepsilon_p(\omega) + \varepsilon_b(\omega,k)$ where ε_p is the plasma dielectric function and ε_b that for the beam. The wave number k is measured in units of $\omega_p/\beta_0 c$. The plasma part is well known:

$$\varepsilon_p(\tilde{\omega}) = \frac{1}{\omega^2}\left(\frac{m_e}{m_i} + \frac{1}{1 + \nu/\omega_p}\right) \quad . \tag{2}$$

The beam part is more complicated. For the 1-D case (propagation along the beam), it is given by

$$\varepsilon_b(\omega,k) = \frac{\alpha a}{\gamma_0^3\,k}\,\frac{k\coth(a) + \omega}{\omega^2 - k^2} + \frac{\alpha a}{2\gamma_0^2\,\sinh(a)}\Big([a(1 - \beta_0^2\omega^2/k^2) - 2\beta_0^2\omega/k]e^{a\omega/k}$$

$$\cdot\ \{E_1[a(\omega/k + 1)] - E_1[a(\omega/k - 1)]\}$$

$$-\ 2\beta_0^2[\sinh(a)(1/a + \omega/k) + \cosh(a)]\Big) \tag{3}$$

where $\beta_0^2 = (1 - 1/\gamma_0^2)$, $\alpha = n_b/n_p$ and $E_1(z)$ is the exponential integral.[6]

For wave propagating obliquely to the beam, (2-D case) the beam contribution is markedly more complicated. We have

$$\varepsilon_b = \frac{-\alpha\beta_0^2}{\gamma_0 k^2} + \frac{\alpha a\beta_0^2}{2\gamma_0 b^2\sinh(a)}\,I(\omega,k,a) \quad , \tag{4}$$

and further

$$I(\omega,k,a) = -\left(\omega^2 + k^2/\beta_0^2\right)\frac{\partial I_0}{\partial \omega} - 2k\omega\frac{\partial I_0}{\partial k} \quad , \tag{5}$$

and

$$I_0 = -\frac{1}{k}\exp\left(\frac{a\,\omega\cos\psi}{k}\right)\cdot\left[F_1(\omega,k,a) + F_2(\omega,k,a)\right] \tag{6}$$

where $\cos\psi = k\ /k$.

F_1 and F_2 are given in terms of tabulated transcendental functions by

$$F_1 = -E_1(a\bar{y}) + E_1(-ax_a) + \sum_{n=1}^{\infty}\frac{(1)(3)\cdots(2n-1)}{(2)(4)\cdots(2n)}\left\{E_{2n+1}(a\bar{y}) - \left(\frac{\bar{y}}{x_a}\right)^{2n}E_{2n+1}(-ax_a)\right\} - i\pi\,\mathrm{sgn}[\mathrm{Im}(a\bar{y})]I_0(-a\bar{y}) \tag{7}$$

where $\bar{y} = (-\ \omega^2/k^2)^{\frac{1}{2}}\sin\psi$,

$$x_a = -\left(1 + \frac{\omega\cos\psi}{k}\right) \quad ,$$

$E_n(z)$ is the exponential integral of n'th order[7] and I_0 is the modified Bessel function of the first kind and zero order. The last term provides the analytic continuation of ε_b into the lower half ω-plane and the values of the exponential integrals are on their principle branch, $-\pi < \arg(z) \leq \pi$. We remark that the Landau-like integrals which give rise to the functions F_1 and F_2 (given below) have been considered by direct numerical integration in Ref. 6. The singular nature of the integrals makes numerical integration very difficult and the analytic continuation to the lower half plane could not be easily extracted. The representation of F_2 depends on the value of $\mu \equiv x_b/\bar{y}$, where $x_b = 1 - \omega/k\cos\psi$.

For $|1 - \mu| < 2$, we have

$$F_2 = \frac{ie^{a\bar{y}}}{(2a\bar{y})^{\frac{1}{2}}}\left\{\gamma[\tfrac{1}{2},a(y - xb)] + \sum_{n=1}^{\infty}\frac{(1)(3)\ldots(2n-1)\ \gamma(n+1/2,\ a(\bar{y} - xb)}{(2)(4)\cdots(2n)(2a\bar{y})^n}\right\} \quad , \tag{8}$$

where $\gamma(a,z)$ is the incomplete gamma function.[8] If $|x_b/\bar{y}| > 1$, then

$$F_2 = -E_1(-ax_b) + E_1(-a\bar{y}) + \sum_{n=1}^{\infty} \frac{(1)(3)\ldots(2n-1)}{(2)(4)\cdots(2n)}$$

$$\times \left[E_{2n+1}(-a\bar{y}) - \left(\frac{\bar{y}}{xb}\right)^{2n} E_{2n+1}(-ax_b)\right] \quad . \tag{9}$$

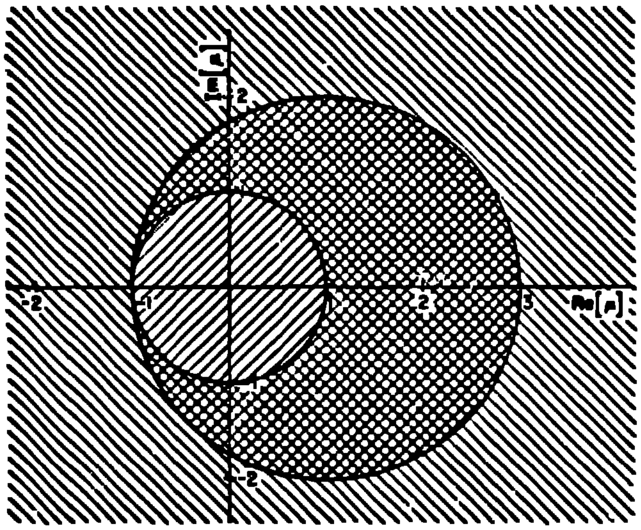

Fig. 1

Domains of validity in the complex μ-plane of the two representations of F_2. Eq. 8 ////, Eq. 9 \\\\\\\\, both: XXX.

The domain of validity of the two representations are illustrated in Fig. 1. There is a finite region of overlap in the plane deleted of the point $\mu = -1$. It can be shown that the choice of domain of ψ above and the definitions of x_b and $\bar{y}$ preclude the value of $\mu = -1$. There is a choice of branch implicit in Eq. (8). The choice is made by requiring that the values of F_2 and its derivative be the same in both representations in the overlap region.

In general, the dispersion relation must be solved numerically; however, in the 1-D case, it is easy to solve for the lowest order correction to the cold beam dispersion relation. This limit is defined by $a > 1$, $a\delta > 1$ where δ is a typical growth rate (in units of ω_p). The dispersion relation is in this case

$$1 - \frac{1}{\omega^2(1 + \nu/\omega)} - \frac{\alpha}{\gamma_0^2 k^2} \frac{1}{(\omega/k - 1)^2} + \frac{2\alpha}{\gamma_0^3 k^2 a} \frac{1}{(\omega/k - 1)^3} \quad . \tag{10}$$

In the collision-free case ($\nu = 0$), this equation has the solution for the growth rate

$$\tilde{\delta} = \frac{e^{2\pi i/3}}{2^{1/3}} \left(1 - \frac{2^{1/3} e^{4\pi i/3}}{3} \tilde{\eta} - \frac{2^{4/3} e^{-2\pi i/3}}{3}\right) \tag{11}$$

where $\quad \tilde{\delta} = \left(\frac{\alpha}{\gamma_0^3}\right)^{-1/3} \delta$

$$\tilde{\eta} \equiv \left(\frac{\alpha}{\gamma_0^3}\right)^{-1/3} \eta = \left(\frac{\alpha}{\gamma_0^3}\right)^{-1/3} \left(k - 1\right) \quad \text{and}$$

$$\epsilon = \left[a\left(\frac{\alpha}{\gamma_0^3}\right)^{1/2}\right] \quad .$$

In the collisional case, the growth rate becomes

$$\tilde{\delta} = e^{3\pi i/4}\left[1 + i\left(\frac{\alpha}{\gamma_0^3 \nu}\right)^{1/2} \frac{1}{\nu} \left(\tilde{\eta} + e^{3\pi i/4}\right) - e^{-3\pi i/4}\epsilon\right] \tag{12}$$

where now

$$\tilde{\delta} = \left(\frac{\alpha}{\gamma_0^3 \nu}\right)^{-1/2} \delta$$

$$\tilde{\eta} = \left(\frac{\alpha}{\gamma_0^3 \nu}\right)^{-1/2} \eta \quad \text{and}$$

$$\epsilon = \left[\left(\frac{\alpha}{\gamma_0^3 \nu}\right)^{1/2} a\right]^{-1} \quad .$$

In the limit $a \to \infty$, the cold beam case, the results above reduce to the well-known growth rates.[9] In both the collisionless case and collisional case, we find the important result that there are no $O(\bar{\theta}^2)$ corrections to the cold beam growth rate. Thus, to have corrections of this order appearing in the growth rate, we must have $a\delta \approx 1$ at least. This is consistent with our recent results in the interpretation of the Lawrence Livermore Laboratory Astron experiments.[10] Our requirement that $a\delta \approx 1$ implies a scattering angle, θ, of about 60 mrad for the effects of scatter to become important. The Astron beam had a $\bar{\theta}$ of ~ 15 mrad and this is consistent with our findings presented in Ref. 10 that the beam was behaving essentially as a cold beam.

III. NUMERICAL SOLUTIONS

In order to obtain results for general values of the parameters, the dispersion relation must be solved numerically. In the 1-D case, we have evaluated the collision frequencies necessary for stabilization of the two-stream mode over a wide range of parameters and compared these with the early results. The scaling over two orders of magnitude in γ_0 are indicated in Table I. The beam-to-plasma density ratio has been held fixed at 10^{-2}, a reasonably typical value

TABLE I

STABILIZING COLLISION FREQUENCY FOR
TWO-STREAM INSTABILITY IN A SCATTERED BEAM

Relativistic Factor, γ_o	Mean Scattering Angle, $\bar{\theta}$	Beam-to-Plasma Density Ratio, n_b/n_p	Singhaus Collision Frequency,[a] ν_s	Stabilizing Collision Frequency,[b] ν_c
11	9.29°	0.01	3.3×10^{-2}	8.7×10^{-2}
100	3.10°	0.01	3.6×10^{-3}	5.8×10^{-2}
1000	0.98°	0.01	3.6×10^{-4}	4.7×10^{-2}

[a]Collisions to stabilize based on Singhaus theory $\nu_s = 3.04/\gamma_o^3\ n_b/n_p\ \beta_o^2/\bar{\theta}^4$.
[b]All frequencies are in units of the plasma frequency.

and the scattering angle has been taken to be the betatron angle. The difference in the scaling of the critical collision frequency between our results and those based on the earlier models is significant.

The mechanism of the collisional stabilization is illustrated in the plot of growth rate as a function of collision frequency presented in Fig. 2. The parameter values are typical of the Ion Physics FX-25 diode beam.[4] The mean scattering angle is that to be expected from Moliere scattering in a 1 mil Ti anode foil. The results are qualitatively the same for other parameter values if the mean scattering is such that the instability in the absence of collisions is in the quasi-hydrodynamic regime.[6] We observe that the growth rates are only weakly affected by collisions until the collision frequency approaches the critical collision frequency at which point the growth is quenched very rapidly. Physically, the phase velocity is moved into the body of the distribution by the collisions and the instability then becomes kinetic in character. Only when it is kinetic do the collisions overcome the growth of the mode. We have recently made progress in solving the 2-D dispersion

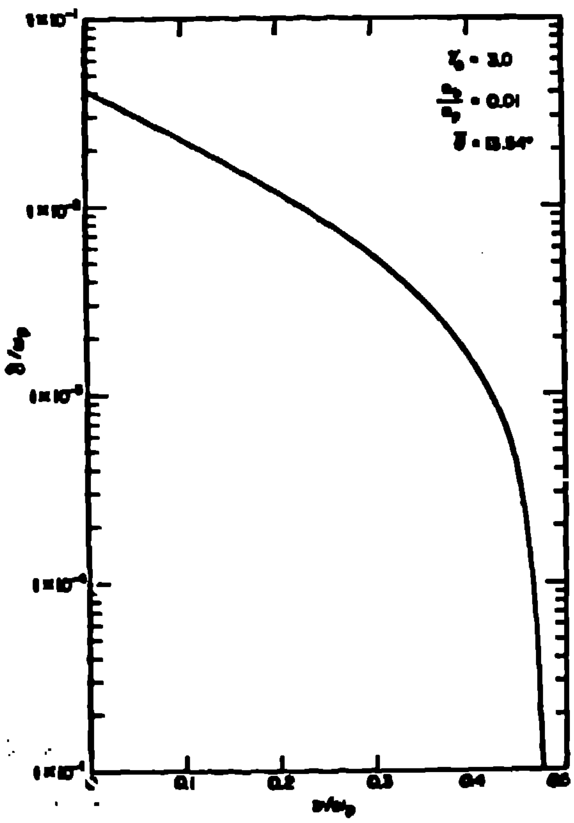

Fig. 2

Instability growth rate δ as a function of plasma electron collision frequency.

relation given in Sec. II. The results are consistent with this interpretation. We find that at small collision frequencies, the off-axis modes have larger growth rates out to some value of propagation angle. As the collision frequency is increased, this cone of angles decreases. When the critical collision frequency is approached, the cone is essentially of zero width with only the parallel mode remaining. This is characteristic of the kinetic regime where the parallel mode has the largest growth rate.[6] It must be emphasized, however, that the 2-D results are of a very preliminary nature and a thorough study of the instability in this case remains to be carried out.

IV. CONCLUSION AND SUMMARY

We have made significant progress in the understanding of the collisional stabilization of the high-frequency two-stream instability in a relativistic electron beam through a rigorous Vlasov treatment.

Substantially different scaling with parameters are obtained in comparison with previous, essentially phenomenological, models and we have gained insight into the mechanism of the stabilization process. Further investigation of the 2-D case remains to be done. The issue of stabilization of the two-stream instability in finite geometry is as yet unexplored as is that of nonlinear saturation, by trapping, of the instability in the presence of momentum spread on the beam and collisions on the plasma electrons. These are important in the application of relativistic electron beams to the heating of dense plasmas[2,11] and possibly in the interpretation of neutral gas propagation experiments.[10] Extension of our research into these areas will necessarily rely on the investigations outlined above.

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